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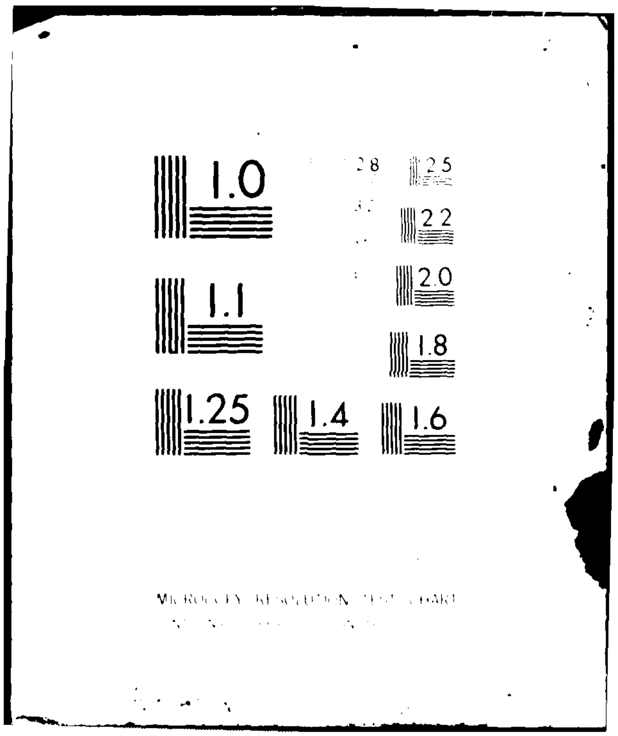
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THESIS

AN EVALUATION OF ESTIMATORS FOR RECEIVER
DETECTION PROBABILITIES
AND UNKNOWN SIGNAL POPULATION SIZE

by

David Scott Hendrickx

September 1981

Thesis Advisor: Donald R. Barr

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An Evaluation of Estimators for Receiver Detection Probabilities and Unknown Signal Population Size

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ABSTRACT

Imagine a signal acquisition system composed of a number of receivers or sensors concurrently scanning the same domain for signals. It is reasonable to expect that different signals will each be detected by a different subset of receivers over the scanning period. Using the data collected from the receivers, it is possible to estimate the total signal population size including those signals not detected by any receiver. Additionally, it is possible to estimate the probability each individual receiver detects signals. Several estimators are developed for these quantities in the context of a model designed to represent the signal detection process. This model forms the basis for a simulation conducted to analyze the behavior of the estimators over a variety of conditions.

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I. INTRODUCTION

Imagine transmission devices emitting signals in the electromagnetic spectrum at unknown times and of unknown quantity. Further, imagine there are receivers concurrently scanning the spectrum in an effort to detect the transmissions. Suppose there are k receivers in the system and a record is kept of each signal detected by each receiver. It is reasonable to expect that some signals will be detected by only one receiver, some by more than one receiver, and some signals will not be detected at all. Assume each signal is identifiable by its characteristics making it possible to determine whether detections by several receivers were actually detections of a common signal. The purpose of this thesis is to demonstrate how data collected by the acquisition system can be used to estimate the total number of signals exposed to the system including the number of signals not detected by any receiver. It would, additionally, be interesting to know the probability each receiver in the system detects signals, and this problem will also be discussed.

It would stand to reason that the detection process is some function of the receiver's capability to detect a signal as well as the signal's ability to be detected.

One such functional relationship (see Barr [Ref. 1]) will be proposed in the context of a model developed to represent the signal detection process. Various estimation principles such as maximum likelihood, method of moments, and least squares will be used to develop estimators for the total signal population size and the receiver intercept probabilities.

A development of several estimators will be shown in detail and their applicability to this problem discussed. These estimation procedures will then be incorporated into a general model constructed to represent the detection process. This model supports the conduct of a computer simulation which also includes the generation of signals with varying signal strength and simulates detections of the signals by the receiver system. The resulting estimates are compared to the actual number of signals generated and the expected receiver intercept probabilities to provide some insight on the quality of the estimators. The estimators are exposed to various conditions of the signal process and their behavior analyzed.

II. GENERAL MODEL

In a statistical approach to modelling the physical phenomenon of signal detection, it is desirable to develop a model of sufficient generality such that the factors associated with radio and radiowave theory can be broadly parameterized. This generality has a distinct advantage in that it is not necessary to precisely define what type of sensor is being used nor what necessarily constitutes a signal. A detection is presumed to be some interception of a signal by a sensor which enables the signal to be identified. It may be that a signal, once detected, is lost and then redetected. This model similarly presumes that a reoccurence of a signal may also be identified. It is also desirable to prescribe a uniform method for counting or indexing detections in a manner which is conveniently applied to mathematical formulae and computer code.

A. SIGNAL DETECTION

It is possible to postulate functional values for the probabilities of signal detection letting π_{ij} be defined to be the probability that receiver i detects signal j, independent of other signals. Each receiver or sensor, regardless of similarities to one another, will be peculiar in its relative ability to detect a given signal. Each signal

j has certain characteristics such as intensity or frequency which affect its ability to be detected. (Depending on the circumstances, a signal may have a variety of identifiable characteristics. For the purpose of this report, these characteristics are considered to be aggregated and called the "strength" of the signal.) Let the random variable V be defined as some measure of these characteristics where $0 < V < 1$. Let $T_i(V)$ represent a specific function that receiver i is "able" to detect a random signal. Thus, a general expression, $\pi_{ij} = T_i(v_j)$, relates the ability of receiver i to detect a signal, to the ability of signal j to be detected. For any future random signal V, $\pi_i = T_i(V)$.

The form of the function considered in this model is $\pi_i = T_i(V) = l_i \cdot V$, where l_i denotes the proportion of signals of strength V that receiver i would detect. It is easily seen that if all signals were certainly detectable, V would take on the value one and $T_i(v) = l_i$. Similarly, if all receivers were of equal specification and certainly able to detect a given signal for any v, $l_i = 1$ and $T_i(v) = v$ for all i. Various conditions on l_i and distributions for V will be considered.

B. METHOD OF INDEXING

Consider that over some specified time, s (unknown) signals will be scanned by k receivers in a signal intercept

system. For any given signal, define the random variable Z to be a k-component vector whose components consist of $z_i = 0$ if receiver i did not detect the signal and $z_i = 1$ if receiver i did detect the signal. Thus, Z_i is a Bernoulli random variable. Let S denote the sample space of Z, so S contains 2^k k-dimensional vectors of zeros and ones. This naturally includes the zero vector which contains the unobservable outcome that no receiver detected the signal.

Over some specified amount of time, for each $z \in S$, let n_z denote the number of times that outcome $Z = z$ occurs and let n be a 2^k-dimensional vector whose components are the n_z's. For example, in a $k = 2$ receiver system, $S = \{(0,0), (1,0), (0,1), (1,1)\}$. The first element, the zero vector is an unobserved signal. The second element represents a signal detected by receiver 1 only, the third by receiver 2 only, and the last represents a signal detected by both receivers. Any given detected signal will appear as an outcome on $Z \in S$. Over many signals, $n_{z=(1,0)}$ is the number of times that receiver 1 only detected the signals. The sum over all n_z will be the total number of signals detected by the acquisition system. Because $n_{z=(0,0)}$ is the number of signals not detected, $\sum_{z \neq 0} n_z \leq s$, where s was the total unknown number of signals present. Let N be the random vector with outcome n. It follows that N is distributed according to the multinomial law, $N \sim M_{2^k}(s,p)$, where p

is the 2^k-dimensional vector denoting the probabilities that a type z outcome occurs.

III. ESTIMATION PROCEDURES

Several procedures are considered for estimating the total number of signals which pass through an acquisition system. Estimators for receiver intercept probabilities are developed for each approach. The maximum likelihood approach uses the properties of the multinomial distribution in conjunction with numerical methods for maximizing the likelihood function. A method of moments procedure is developed which finds a solution vector to a set of simultaneous equations based on the binomial characteristics of each receiver. Finally, a least squares approach, again with numerical methods, minimizes the square error between actual and expected detection outcomes for find estimates of signal population size and receiver intercept probabilities.

A. MAXIMUM LIKELIHOOD/CONDITIONAL MAXIMUM LIKELIHOOD

The concept behind this procedure is one of estimating the size of a multinomial population with incomplete observations. This technique follows an approach discussed by Sanathanan [Ref. 2]. It is convenient, in the statistical sense, to imagine a set of cells representing the 2^k elements in the sample space S described in section II. A detection by some subset of sensors in the k-receiver system

adds a count of one to the appropriate cell. Recall that after many signals, the total counts in each cell will comprise the 2^k-dimensional vector N. N is distributed multinomial,

$$N \sim M_{2^k}(s,p).$$

Suppose the ordering within the vector N is such that the number of signals which missed detection is placed cell n_o and has probability p_o. Let r denote the number of remaining cells, $r = 2^k-1$. Define $t = \sum_{i=1}^{r} n_i$, thus $n_o = s-t$. The observation of $(n_1,\ldots,n_r)$ yields the likelihood function L,

$$L(s,p) = \frac{s!}{n_1!\ldots n_r!(s-t)!}(p_1)^{n_1}\ldots(p_r)^{n_r}(p_o)^{s-t}.$$

L(s,p) may also be written as the product

$$L(s,p) = L_1(s,p)L_2(p)$$

where

$$L_1(s,p) = \frac{s!}{t!(s-t)!}(1-p_o)^t(p_o)^{s-t}$$

$$(1) \qquad L_2(p) = \frac{t!}{n_1!\ldots n_r!}(q_1)^{n_1}\ldots(q_r)^{n_r}$$

with

$$q_i = p_i/(1-p_o), \; i = 1,\ldots,r.$$

L_1 is the likelihood based on the probability of t and hence L_2 is the likelihood based on the conditional probability of $(n_1,\ldots,n_r)$ given t. The following lemma is known (see e.g. Chapman (1951)).

LEMMA: For any given p, $\hat{s} = (t/(1-\hat{p}_o))$ (greatest integer $(t/1-\hat{p}_o)$) maximizes $L_1(s,p_o)$, where $L_1(s,p_o)$ is defined above. If $1-\hat{p}_o = t/\hat{s}'$ for some integer $\hat{s}'$, then $\hat{s}$ and $\hat{s}-1$ both maximize $L_1(s,p_o)$. Otherwise, $\hat{s}$ is the unique maximum.

Since $\sum_{i=0}^{r} p_i = 1$, the sum over p_i, $i = 1,\ldots,r$ will uniquely specify p_o. Hence, to find an estimate of s, it is only necessary to maximize the conditional likelihood function $L_2(p)$. More conveniently, the maximum of the log-likelihood function is found. Equation (1) may be transformed as follows:

$$(2) \quad \log(L_2(p)) = \log(K) + n_1\log(q_1) + \ldots + n_r\log(q_r),$$

where $K = t!/(n_1!\ldots n_r!)$.

Equation (2) can be rewritten as

$$\log(L_2(p)) = \log(K) + \sum_{i=1}^{k} m_i\log(\pi_i) + \sum_{i=1}^{k}(t-m_i)\log(1-\pi_i) - (s-t)\log(1-(1-\prod_{i=1}^{k}\pi_i)),$$

where m_i denotes the sum of cell counts in S for which receiver i had detections (i.e. receiver i only detected, receiver i and receiver j both detected, etc.) and $t-m_i$ represents the number of non-detects by receiver i. Thus, it is possible to solve directly for estimates of the receiver intercept probabilities, $\underline{\pi}$, without solving for the 2^k components of p.

Since K does not depend on p, it is not necessary to include log(K) when maximizing (2). Given that each value

of n_i, $i \neq 0$ is observable, it is possible using a *numerical* method such as the generalized reduced gradient to maximize the function for optimal values of π_i. The value for p_o is clearly $\hat{p}_o = \prod_{i=1}^{k}(1-\hat{\pi}_i)$ and from the lemma, $\hat{s} = t/(1-\hat{p}_o)$.

B. METHOD OF MOMENTS

As previously defined, m_i denotes the total number of detections made by receiver i. Since each detection can be considered a Bernoulli trial, the random variable M_i is distributed Binomial(s,π_i). For a k-receiver system

$$m_1 = \hat{s}\,\hat{\pi}_1$$

$$m_2 = \hat{s}\,\hat{\pi}_2$$

$$\vdots$$

$$m_k = \hat{s}\,\hat{\pi}_k\ .$$

The parameters s and π_i are all unknown, hence k + 1 unknowns exist in the above k equations. To obtain a k + 1st equation, consider all the signals which were detected by two or more receivers in the system. Define

$$m^* = \sum_{i<j} (\text{number of signals detected simultaneously by receivers } i \text{ and } j).$$

If independence between receivers is assumed

$$E(m^*) = s \sum_{i<j} \pi_i \pi_j$$

$$= s \sum_{i<j} (m_i m_j / s^2)$$

$$= \frac{1}{s} \sum_{i<j} m_i m_j\ .$$

The estimators for s and π_i thus become $\hat{s} = (\sum_{i<j} m_i m_j)/m^*$ and $\hat{\pi}_i = m_i m^* / \sum_{i<j} m_i m_j$.

C. LEAST SQUARES

Define A to be the event that one or more receivers detect a given signal. Hence, P(A) is the probability that the acquisition system will successfully intercept any given signal. Now, m_i is defined to be the number of events for which receiver i had detections, and t, the total number of detections by the system. Following a procedure developed by Knorr [Ref. 3] , it is possible to postulate an estimator for π_i as follows:

$$\hat{\pi}_i = (m_i/t) \cdot \hat{P}(A) \ . \tag{3}$$

In this expression, m_i/t denotes the proportion of signals detected which were observed by receiver i.

Define $z_.$ to be the number of receivers which detect a signal,

$$z_. = \sum_{i=1}^{k} z_i \qquad 0 \leq z_. \leq k \ .$$

A value for $z_.$ can be assigned for each signal intercepted. Further, let I^j represent the number of occasions where $z_. = j$, and let $P_{z_.}$ be the mass function for $z_.$. It follows that $P_{z_.}(j)$ will be the probability that exactly j receivers will detect a given signal. A possible estimator for $P_{z_.}$ is

$$\hat{P}_{z_.}(j) = (I^j/t) \cdot \hat{P}(A) \ . \tag{4}$$

Note that for j = 0,

$$\hat{P}_{z_.}(0) = 1 - \hat{P}(A) \ .$$

Both estimators for π_i and $P_{z.}(j)$ rely on estimating the value of P(A), where $0 \leq P(A) \leq 1$. By setting a value to P(A), say $\hat{P}(A)$, and given values for m_i, I^j, and t, one can find $p_{z.}(j)$ as a function of $\hat{P}(A)$, using (4). Using the same value for P(A), one can similarly compute $\hat{\pi}_i$ using (3). It is possible to find a similar value for $\hat{P}_{z.}(j)$, denoted as $\tilde{P}_{z.}(j)$, by making use of its functional relationship to $\hat{\pi}_i$. Knorr suggests the following algorithm to compute $\tilde{P}_{z.}(j)$:

1. Calculate $q_i = \hat{\pi}_i/(1 - \hat{\pi}_i)$
2. Expand $\prod_{i=1}^{k} (x + q_i)$ to obtain the polynomial

$$a_o x^k + a_1 x^{k-1} + \ldots + a_k$$

3. Calculate $\tilde{P}_{z.}(j) = a_n/\sum_{j=0}^{k} a_k$.

Now form the square error of P(A), where

$$E^2(P(A)) = \sum_{j=0}^{k} (\hat{P}_{z.}(j) - \tilde{P}_{z.}(j))^2. \tag{5}$$

By varying $\hat{P}(A)$ over its range, it is possible to find a "least squares" estimate, $0 \leq \hat{P}(A) \leq 1$, which minimizes the above expression. Using $\hat{P}(A)$, $\hat{\pi}_i$ can be found using (3). The total signal population size passing through the acquisition system is estimated by $\hat{s} = t/\hat{P}(A)$.

IV. THE SIMULATION

Three methods have been discussed which provide estimators for determining receiver intercept probabilities. Similarly, estimators are found for the total number of signals passing through an acquisition system given only a portion of the signals were detected. In this section, the techniques used to test each method will be discussed.

The three methods of analysis were used as the basis for a model developed to support a computer simulation. The model was designed to perform two major functions necessary for the simulation. The first function is to generate signals simulating the transmission of signals from some source. The second function of the model is to statistically relate each signal to each receiver to determine actual detections. During the simulation, each receiver must either detect or not detect a given signal. Thus, an accounting procedure was developed relating each receiver to each signal. Finally, the model applies each method of analysis to the data generated and estimated values of intercept probabilities, $\hat{\pi}_i$, and total signals, $\hat{s}$, are compared to the $E(\pi_i)$, and the number of signals specified to the signal generator.

The simulation is performed in two parts. The first consists of one replication of each method in each of 72 different situations. A situation or environment is

comprised by varying

1. The total number of signals generated.
2. The number of receivers in the acquisition system.
3. The parameters specifying the relative "ability" of a receiver to detect a signal.
4. A parameter specifying the relative "ability" of a signal to be detected.

The second part of the simulation replicates twelve of the above environments 100 times each, thus enabling an analysis of mean absolute error, bias, and variance of the estimators.

A. INPUT: "MAIN ROUTINE"

The main routine reads all required data and subsequently calls each subroutine in sequence. Input requirements for the simulation include the random number generator seed, the number of signals to be generated, the number of receivers in the system, each receiver's ability to detect a signal of strength V, the parameter specifying the "ability" of a signal to be detected, and the method of analysis to be used.

Various loops in the main routine can be established to test one or more methods on many data sets (environments) or to replicate over one data set.

B. SIGNAL GENERATION: "SUBROUTINE GNRATE"

Recall that $T_i(V)$ represents the specific function that receiver i is able to detect a random signal and that the form of the function used (see II. A.) in this model is

(6) $$T_i(V) = l_i \cdot V \ .$$

In the simulation, l_i is a user input denoting elements of the probability vector specifying the ability of receiver i to detect any given signal. Now, suppose V is a Beta(α,1) random variate denoting the ability of a signal to be detected. The Beta distribution was selected due to its ease and simplicity of generation and its desirable behavior as alpha is allowed to vary. All outcomes on V are on the range (0,1) and can be envisioned, under certain conditions, to be the "probability" that a random signal v is "detectable". Furthermore, it is possible to imagine a case where V is degenerate at 1 denoting that signal v is certainly detectable. Clearly, when $\alpha = 1$, v becomes a uniform(0,1) outcome. This implies that the "ability" of signal j to be detected is random with an equal likelihood of being undetectable as detectable. Thus, by varying the α parameter of a Beta(α,1) random variable, it is possible to specify to what degree each signal is expected to be "detectable".

For integer $\alpha \leq 1$ and $\beta = 1$, the Beta distribution has the following cdf:

$$F(v|\alpha,1) = v^{\alpha}, \qquad 0 < v < 1 \ .$$

Since all values of v^{α} lie on the interval (0,1), it is a simple matter to generate random Beta (α,1) variates using the probability integral transform of $V^{\alpha} \sim U$. Let U be a uniform(0,1) random variable and V^{α} an outcome on U.

Then,

$$V^{\alpha} = U$$

and

$$V = U^{1/\alpha}$$

Because the $\lim_{\alpha \to \infty} u^{1/\alpha} \to 1$, for a sufficiently large , the Beta(α,1) distribution can, in general, be used to represent the case where $\pi_i = l_i$. An α value of 100 was selected to simulate a signal which is virtually certain to be detectable. An intermediate case of $\alpha = 10$ is also tested.

Each signal v_j is therefore defined to be an outcome on V_j. In the physical sense, $\alpha = 1$ represents a system where all receivers, in some way, act alike or show dependency. For example, if an outcome on $V \sim U(0,1)$ is a low value, nearly all the receivers in the system, regardless of their ability to detect the signal, will be restricted from doing so simply because the signal is not readily detectable. Any outcome on V has this affect on the entire system, thus causing the receivers to show interdependence. On the other hand, if V is degenerate at 1 (simulated by a large alpha value), the detection of the signal is based solely on the characteristics of the receiver. In this manner, the receivers tend to act independently of one another. Hence, changes in alpha, in a restricted sense, give some indication of correlation through receivers. The case where $\alpha = 10$ demonstrates an intermediate degree of dependence between receivers.

In this model, each signal v is scanned by each receiver. As previously mentioned, a function of the model is to statistically relate each receiver to each signal to determine actual detections. $T_i(v)$ is computed for each receiver using (6) and represents the probability that receiver i detects signal j. To determine if the detection occurred, $T_i(v)$ is compared against a uniform(0,1) random number, where if

$$U(0,1) \quad T_i \leq \text{detection occurs}$$

$$U(0,1) \quad T_i > \text{no detection occurs.}$$

Recall that Z is a k-component vector of zeros and ones denoting which receivers did or did not detect a given signal. Hence, Z = z is an outcome on the k comparisons above, and becomes the element in the sample space S which specifies which combination of receivers detected the signal.

C. COUNTING: "SUBROUTINE COUNT"

The estimators developed in section III rely on certain "counts" of events related in various fashions to the receiver system. Subroutine COUNT takes the data created by the signal generator and performs the necessary accounting. Subroutine GNRATE passes the vector N to subroutine COUNT. (N is the 2^k-dimensional vector containing elements n_z, n_z being the number of times, out of s trials, that z, $z \epsilon S$, occurs.)

Let B denote an index of the elements in the vector N. Suppose B = 4 represents both receivers 1 and 2 detecting a common signal, and over many signals this combination occurs twelve times. This event is then annotated as N(4) = 12. Note that N(1) is the zero vector and cannot be observed. A convenient method of arranging B is to consider the binary number system. Figure 1 demonstrates how the binary numbers can be used to order B for a 4-receiver system. For example, if receivers 1, 2, and 3 all detect a common signal, this is associated with the binary number 0 1 1 1 and the related value of B is the base ten equivalent, 7. B relates directly to the random vector Z. By computing a value for B, Z can be obtained simply by computing the binary equivalent of B. One convenient algorithm for computing an outcome on Z is as follows:

1. Let B = 1
2. Draw a U(0,1) random number
3. If $U(0,1) \leq T_i$, let $B = B + 2^{(i-1)}$
4. If $i \leq k$, go to 2 and repeat
5. Let $z = \text{base}_2 B$.

Using Figure 1 as a reference, it can easily be shown how the necessary counting is performed. Notice that the array of binary numbers is actually a reflection of the way each receiver is listed by the index. By considering the binary array to be a (2^k X k) matrix, various row and column sums can be used as multipliers in the counting procedure.

N(B)	Index	Receiver i, i=1,4 1	 2	 3	 4	Associated Binary Number
N(1)	0					0 0 0 0
N(2)	1	1				0 0 0 1
N(3)	2		2			0 0 1 0
N(4)	3	1	2			0 0 1 1
N(5)	4			3		0 1 0 0
N(6)	5	1		3		0 1 0 1
N(7)	6		2	3		0 1 1 0
N(8)	7	1	2	3		0 1 1 1
N(9)	8				4	1 0 0 0
N(10)	9	1			4	1 0 0 1
N(11)	10		2		4	1 0 1 0
N(12)	11	1	2		4	1 0 1 1
N(13)	12			3	4	1 1 0 0
N(14)	13	1		3	4	1 1 0 1
N(15)	14		2	3	4	1 1 1 0
N(16)	15	1	2	3	4	1 1 1 1

Figure 1• Binary Counter for a 4-Receiver System

The total number of signals detected, t, is the sum over elements $n_1,\ldots,n_r$ in the vector N. The number of signals detected by receiver i, m_i, can be computed by looking in the appropriate column in the binary matrix. Receiver 1 is represented in the array by the far right column. The value for m_1 is computed by summing those elements in the vector N for which a 1 appears in the far right column. This procedure is repeated over all k columns.

The value of m^*, $m^* = \sum_{i<j}$ (the number of signals detected by receivers i and j simultaneously), is obtained by summing those elements in N which contain two or more ones in the associated binary number. Remember in computing m^*, that if, for example, three receivers detect a common signal, then that element of N will be counted three times. Likewise, in a four receiver detect, that element of N will be counted six times. Hence, depending on the number of ones in the associated binary number, there is an appropriate multiplier assigned to each element of N.

The value for $\sum_{i<j}(m_i m_j)$ is the sum over the totals obtained in each column, done for all $i < j$.

D. COMPUTING ESTIMATES: "SUBROUTINE SOLVE"

Subroutine COUNT passes all the necessary accounting information to subroutine SOLVE which performs the operations and optimizations required to produce estimates of

π_i and s as discussed in section III. Using the input parameter METHOD in the main routine, it is possible to specify which one or more methods to be used.

1. Method 1: Maximum Likelihood

Maximum likelihood estimators will be found using numerical methods because as k increases, Lagrangian and other techniques quickly become too burdensome. In this simulation, the non-linear maximization approach used is the Generalized Reduced Gradient Technique (see Lasdon, [Ref. 4]). This computer method is available in most computer libraries and is easy to use.

Library routines DATAIN, GRG, and OUTRES are called in sequence. Routine DATAIN reads all necessary data while routine GRG performs the optimization. It is necessary for the user to supply a subroutine GCOMP which evaluates constraint and objective function values (see Appendix A). Routine OUTRES provides the user information on each iteration of the maximization. The quantity of output can be varied by a user inputted flag in DATAIN. In this simulation, the iterative data is totally suppressed.

2. Method 2: Method of Moments

This method is perhaps the only one of the three discussed which can be computed by hand. Estimates for π_i and s are simple functions of the data provided by subroutine COUNT and are computed as discussed in section III.

3. Method 3: Least Squares

Again, numerical techniques are necessary in solving least squares estimators. An initial value of P(A) = .1 begins the least squares minimization. The value .1 is chosen because of the potential presence of trivial solutions when the system intercept probability falls within the interval [0,.1) (see Knorr, 1979). P(A) is incremented in steps of .01 on the interval [.1,1]. For each value of P(A), $\hat{P}_{z.}(j)$ is computed using (4) and $\tilde{P}_{z.}(j)$ is numerically evaluated using the algorithm on page 19. The value of P(A) which minimizes the least squares expression (5) becomes the system intercept probability and is used to compute $\hat{\pi}_i$ and $\hat{s}$.

E. *OUTPUT: "SUBROUTINE OUTPUT"*

Two forms of subroutine OUTPUT are used depending on which part of the simulation is being performed. Recall that in the first part of the simulation, 72 environments were examined one time each to obtain a feel for the behavior of the estimators. A sample listing of the results is shown at Figure 2.

The second part of the simulation focuses on analyzing the mean absolute error (MAE), bias and variance of the estimators for π_i and s. Twelve of the environments were selected for analysis and replicated 100 times each. For each replication, the MAE of the element in the vector $\underline{\hat{\pi}}$

was computed by comparing the estimated values to the expected values of $\underline{\pi}$. The expected value of π_i is easily computed given l_i, the specified ability of receiver i to detect a signal of strength V, and the distribution of V. Given that the form of the function for $\pi_i = T_i(V) = l_i \cdot V$, $E(\pi_i) = E(l_i \cdot V) = l_i \cdot E(V)$. The expected value of a Beta$(\alpha,1)$ random variable can be shown to be $\alpha/(1+\alpha)$. Hence, $E(\pi_i) = l_i \cdot \alpha/(1+\alpha)$. The MAE of the receiver intercept probability becomes $ABS(\hat{\pi}_i - E(\pi_i))$. The MAE's are summed across the entire probability vector to produce an MAE estimate for the system. The 100 system MAE estimates are then sectioned into ten blocks of ten replications each, making use of the computer library routine SECTN. In this manner, estimated sample parameters on the system MAE (i.e. mean, variance, etc.) are computed along with estimated parameters of the sample parameters. Figure 3 shows a typical output listing for subroutine SECTN. (Remember that these sample parameters are on the estimate of the MAE for $\hat{\underline{\pi}}$, not on the estimate of $\hat{\underline{\pi}}$ directly.)

Following the sample parameters is a listing of the initial input to the signal generator. Next, the method of analysis used is shown, followed by direct estimates of the mean and variance for each element in $\hat{\underline{\pi}}$.

The identical procedure is followed for analyzing the estimator $\hat{s}$. A typical output listing for the analysis on

$\hat{s}$ is shown at Figure 4. The above procedure is duplicated for each of the three methods discussed in section III.

```
INPUT PARAMETERS TO SIGNAL GENERATOR
        NUMBER OF SIGNALS GENERATED:     1000
        NUMBER OF RECEIVERS:                3
        ALFA PARAMETER:                 100.0
        RECEIVER PROBABILITY VECTOR:     0.20 0.50 0.80

CELL COUNTS
          N( 1) =   22
          N( 2) =   83
          N( 3) =   14
          N( 4) =  312
          N( 5) =   75
          N( 6) =  324
          N( 7) =   86

OPTIMIZATION COEFFICIENTS
     OBSERVED NUMBER OF DETECTIONS BY RECIEVER I:
          197.0   507.0   797.0
     OBSERVED NUMBER OF NO-DETECTS BY RECIEVER I:
          719.0   409.0   119.0
     OBSERVED NUMBER OF TIMES J RECIEVERS DETECTED A COMMON SIGNAL:
          417.0   413.0    86.0
     NUMBER OF SIGNALS INTERCEPTED:     916.0

OPTIMIZATION METHOD:  MAXIMUM LIKELIHOOD/CONDITIONAL MAXIMUM LIKELIHOOD

    ESTIMATES OF INTERCEPT PROBABILITIES:
         0.20 0.51 0.80

    ESTIMATE OF THE TOTAL NUMBER OF TRANSMISSIONS:    992.79

OPTIMIZATION METHOD:  METHOD OF MOMENTS

    ESTIMATES OF INTERCEPT PROBABILITIES:
         0.20 0.51 0.81

    ESTIMATE OF THE TOTAL NUMBER OF TRANSMISSIONS:    985.05

OPTIMIZATION METHOD:  LEAST SQUARES

    SYSTEM INTERCEPT PROBABILITY:      0.92

    ESTIMATES OF INTERCEPT PROBABILITIES:
         0.20 0.51 0.80

    ESTIMATE OF THE TOTAL NUMBER OF TRANSMISSIONS:    995.65
```

Figure 2. Sample Output - Form 1

ESTIMATED SAMPLE PARAMETERS

SECTION	MEAN	MEDIAN	VARIANCE	STD. DEV.	COEF VAR	SKEWNESS	KURTOSSIS	MINIMUM	MAXIMUM
1	6.3145E-02	6.7831E-02	2.0316E-04	1.4253E-02	2.2572E-01	-2.0469E+00	3.1451E+00	2.7080E-02	7.6427E-02
2	5.9670E-02	5.7070E-02	1.8705E-04	1.3677E-02	2.2920E-01	7.3038E-01	-6.8179E-01	4.2668E-02	8.6018E-02
3	5.6352E-02	5.7295E-02	3.1110E-04	1.7638E-02	3.1300E-01	8.9677E-02	-7.0264E-01	2.9953E-02	8.7658E-02
4	6.2917E-02	6.1531E-02	9.9502E-05	9.9751E-03	1.5854E-01	7.5177E-01	4.2400E-02	4.8618E-02	8.3410E-02
5	6.9558E-02	6.5162E-02	2.2593E-04	1.5031E-02	2.1597E-01	1.0440E+00	-4.3626E-01	5.2240E-02	9.5775E-02
6	6.8696E-02	6.9629E-02	2.3781E-04	1.5421E-02	2.2448E-01	-1.6892E-03	-9.9306E-01	4.3751E-02	9.4033E-02
7	5.7057E-02	5.2021E-02	3.1973E-04	1.7881E-02	3.1339E-01	2.4016E-02	-4.8188E-01	2.4653E-02	8.5217E-02
8	5.7838E-02	5.9347E-02	2.3780E-04	1.5421E-02	2.6662E-01	-7.6418E-01	7.7217E-01	2.4836E-02	8.2112E-02
9	6.6834E-02	7.1743E-02	3.4254E-04	1.8508E-02	2.7692E-01	-9.9461E-01	5.4297E-01	2.7165E-02	9.2590E-02
10	6.5951E-02	6.6576E-02	1.0618E-04	1.0304E-02	1.5624E-01	9.1694E-01	1.3497E-01	5.4044E-02	8.7759E-02
UNSECTIONED	6.2806E-02	6.2926E-02	2.2928E-04	1.5109E-02	2.4057E-01	-2.2722E-01	2.8916E-01	2.4653E-02	9.5775E-02

ESTIMATED PARAMETERS OF SAMPLE PARAMETERS

PARAMETER	MEAN	MEDIAN	VARIANCE	STD. DEV.	COEF VAR	SKEWNESS	KURTOSSIS	STD. DEV./NS**.5
MEAN	6.2806E-02	6.3031E-02	2.4031E-05	4.9021E-03	7.8052E-02	2.4228E-03	-1.7591E+00	1.5502E-03
MEDIAN	6.2821E-02	6.3346E-02	4.0513E-05	6.3650E-03	1.0132E-01	-2.3956E-01	-1.2988E+00	2.0128E-03
VARIANCE	2.2708E-04	2.3187E-04	6.9194E-09	8.3183E-05	3.6632E-01	-2.3527E-01	-1.1273E+00	2.6305E-05
STD. DEV.	1.4811E-02	1.5226E-02	8.5747E-06	2.9283E-03	1.9771E-01	-5.7377E-01	-9.0578E-01	9.2599E-04
COEF. VAR.	2.3801E-01	2.2746E-01	3.0596E-03	5.5313E-02	2.3240E-01	-1.0831E-01	-1.1500E+00	1.7492E-02
SKEWNESS	-2.4857E-02	5.6846E-02	9.7759E-01	9.8873E-01	3.9777E+01	-9.5131E-01	-2.5488E-01	3.1266E-01
KURTOSSIS	1.3420E-01	-1.9693E-01	1.4433E+00	1.2014E+00	8.9523E+00	1.9737E+00	2.9133E+00	3.7991E-01

NS=# OF SECTS

STATISTICS FOR MEAN ABSOLUTE ERROR ON RECEIVER PROBABILITY VECTOR

INPUT PARAMETERS TO SIGNAL GENERATOR

NUMBER OF SIGNALS GENERATED: 10000
NUMBER OF RECEIVERS: 6
ALFA PARAMETER: 100.0
RECEIVER PROBABILITY VECTOR: 0.20 0.20 0.30 0.30 0.40 0.40

OPTIMIZATION METHOD: MAXIMUM LIKELIHOOD/CONDITIONAL MAXIMUM LIKELIHOOD

AVERAGE VALUE OF RECEIVER PROBABILITIES:	0.2110D+00	0.2109D+00	0.3080D+00	0.3073D+00	0.4028D+00	0.4006D+00
VARIANCE:	0.3659D-04	0.5443D-04	0.3610D-04	0.9812D-04	0.3617D-04	0.2987D-04

Figure 3. Sample Output - Form 2 for Estimates on Receiver Intercept Probabilities

ESTIMATED SAMPLE PARAMETERS

SECTION	MEAN	MEDIAN	VARIANCE	STD. DEV.	COEF VAR	SKEWNESS	KURTOSSIS	MINIMUM	MAXIMUM
1	9.4614E+01	8.7336E+01	1.2624E+03	3.5530E+01	3.7553E-01	7.4346E-01	-7.1681E-01	5.1642E+01	1.5936E+02
2	1.1085E+02	1.1331E+02	1.8597E+03	4.3124E+01	3.8901E-01	1.6917E-01	-5.1215E-01	3.8608E+01	1.8815E+02
3	1.1389E+02	1.2876E+02	2.3945E+03	4.8933E+01	4.2964E-01	-7.2481E-01	-8.8420E-01	2.7756E+01	1.7896E+02
4	1.1540E+02	1.1796E+02	1.8088E+03	4.2530E+01	3.6854E-01	-1.9399E-01	-8.2567E-01	4.1406E+01	1.8430E+02
5	1.1003E+02	1.0164E+02	3.1407E+03	5.6042E+01	5.0935E-01	3.9778E-02	-1.2786E+00	2.6580E+01	2.0000E+02
6	9.8982E+01	1.0225E+02	2.5656E+03	5.0652E+01	5.1173E-01	-5.6854E-02	-8.0012E-02	9.8438E+00	1.9062E+02
7	1.0202E+02	1.0134E+02	4.0390E+03	6.3553E+01	6.2296E-01	6.5030E-01	-2.6053E-01	1.8429E+01	2.2637E+02
8	9.1441E+01	8.9035E+01	2.8567E+03	5.3448E+01	5.8451E-01	-4.2755E-01	-1.3887E+00	8.5009E+00	1.5459E+02
9	1.2987E+02	1.3995E+02	3.0581E+03	5.5300E+01	4.2580E-01	-7.5488E-01	-4.3400E-01	2.2929E+01	2.0191E+02
10	1.1133E+02	1.1716E+02	3.0487E+03	5.5215E+01	4.9597E-01	-1.6784E-01	-1.9666E-01	1.4261E+01	2.0742E+02
UNSECTIONED	1.0764E+02	1.0741E+02	2.4841E+03	4.9840E+01	4.6216E-01	-4.1563E-02	-5.3992E-01	6.5005E+00	2.2637E+02

ESTIMATED PARAMETERS OF SAMPLE PARAMETERS

PARAMETER	MEAN	MEDIAN	VARIANCE	STD. DEV.	COEF VAR	SKEWNESS	KURTOSSIS	STD. DEV./NS**.5
MEAN	1.0784E+02	1.1044E+02	1.2905E+02	1.1360E+01	1.0534E-01	3.6020E-01	-3.5218E-01	3.5924E+00
MEDIAN	1.0987E+02	1.0778E+02	2.8140E+02	1.6775E+01	1.5268E-01	3.7598E-01	-8.8185E-01	5.3047E+00
VARIANCE	2.6034E+03	2.7111E+03	6.4964E+05	8.0600E+02	3.0959E-01	-5.6690E-03	-5.5830E-01	2.5488E+02
STD. DEV.	5.0433E+01	5.2050E+01	6.6613E+01	8.1617E+00	1.6183E-01	-3.8565E-01	-5.5830E-01	2.5809E+00
COEF. VAR.	4.7130E-01	4.6281E-01	7.7571E-03	8.8075E-02	1.8687E-01	4.8531E-01	-1.2276E+00	2.7852E-02
SKEWNESS	-7.2321E-02	-1.1235E-01	2.5562E-01	5.0558E-01	6.9909E+00	3.1430E-01	-9.6238E-01	1.5988E-01
KURTOSSIS	-6.5773E-01	-6.1448E-01	1.9757E-01	4.4449E-01	6.7579E-01	-4.4075E-01	-1.1956E+00	1.4056E-01

NS=# OF SECTS

STATISTICS FOR MEAN ABSOLUTE ERROR ON ESTIMATES OF SIGNALS GENERATED

INPUT PARAMETERS TO SIGNAL GENERATOR

NUMBER OF SIGNALS GENERATED: 10000
NUMBER OF RECEIVERS: 6
ALFA PARAMETER: 100.0
RECEIVER PROBABILITY VECTOR: 0.20 0.20 0.30 0.30 0.40 0.40

OPTIMIZATION METHOD: MAXIMUM LIKELIHOOD/CONDITIONAL MAXIMUM LIKELIHOOD

AVERAGE NUMBER OF SIGNALS ESTIMATED: 0.9894D+04
VARIANCE: 0.2837D+04

Figure 4. Sample Output - Form 2 for Estimates on Signal Population Size

V. ANALYSIS OF RESULTS

The purpose of this section is to demonstrate the results of the simulation described in section IV and to discuss, in general terms, trends and observations about the behavior of the different methods of analysis as they apply to the model. The first part of the simulation used each of the three methods in 72 different environments to obtain estimates of the total signal population size. In the second part of the simulation, twelve environments were replicated 100 times each and mean absolute errors (MAE) of the estimates for radio intercept probabilities, π_i, and total signal population size, s, were obtained. The mean and variance of the MAE estimates were computed over all replications. Additionally, the mean and variance of the estimators themselves were computed.

The data shown in Table 1 and Table 2 summarize the results obtained in the first part of the simulation. In Table 1 are the estimates of signals generated for a 3-receiver system using each of the three methods of analysis. The input values for the radio intercept capabilities and the alpha parameter are listed in the first column. The actual numbers of signals generated are listed in the second column comprising an environment. The 6-receiver intercept systems are shown in Table 2. In the 6-receiver environments,

two receivers share a common value for l_i, thus, a vector of .6/.7/.8 implies that two receivers were specified to have l_i values of .6, two of .7, and two of .8 .

This part of the simulation was designed to give a feel for how the estimators would behave over a wide range of conditions and to help select an interesting set of environments from which to analyze in more detail. Resource limitations precluded a complete analysis of all environments. Those situations indicated with an (*) are the ones selected for replication.

Though no formal claims can be made about the estimators in the first stage of the simulation, a review of the data in Tables 1 and 2 seems to indicate that all the methods perform better as alpha increases (as the receivers become less dependent) and as the receiver intercept probabilities are increased. This result is intuitively appealing because changes in the parameters which increase the ability of a signal to be detected or a receiver to detect a signal will result in an increase in signals detected. As a higher portion of the total signals are detected, more data are available for estimating the total number of signals present.

The results of the second part of the simulation are shown in Tables 3 - 8. Tables 3 - 5 show mean and variance estimates for the receiver intercept probabilities for

alpha equal to 1, 10, and 100 respectively. Tables 6 - 8 show mean and variance estimates for the total signal population sizes. Notice that all mean estimates of receiver intercept probabilities tend to be positively biased. The bias is more significant for alpha equal to 1 and improves greatly for alpha equal to 10, and tends to become negatively biased for alpha equal to 100. Thus, it appears that a high degree of dependence between receivers will have a significant impact on the bias of the estimators. Similar trends are not as readily apparent on mean estimates of population size. Though maximum likelihood and method of moment estimators seem to be negatively biased, the least squares estimator shows several instances of strong positive bias, particularly for small sample size. One possible explanation for this may be found in the large variance estimates for the least squares estimators. This may be an indication that the model is not adequately distinguishing between the trivial and non-trivial solutions for $\hat{P}(A)$. There appears to be consistently much better estimates for signal population size as alpha increases.

With the wide variety of changing parameters and relatively few replicated trials, it is not possible to make comparative claims between the estimators under changing conditions, but only to look at the estimators under the conditions tested. However, because there were four environments tested for each value of alpha, there are some

suggestions about the quality of each estimator which can be made. For each value of alpha, the sum of the means and variances of the MAE estimates was computed over all environments tested for each estimator. These sums represent a score for each estimator over various conditions but with constant alpha. These mean absolute error scores for estimates of radio intercept probabilities and total signal population size are shown in Table 9. Notice that the method of moments estimator appears to perform the best for estimates of π_i when alpha equals 10 and 100. The variances of the estimates are, however, too large to claim it is the best estimator. This table does show that there is a significant reduction in error as receiver dependence decreases. No one estimator appears to be better for signal population size over all tested values of alpha, but again, the estimators do tend to improve as alpha increases. It is interesting to note that maximum likelihood estimates tend to have the smallest variability while the least squares estimates have the largest.

Ideally, all 72 environments should be replicated and estimates compared over all conditions to obtain performance criteria on each estimator. A data analysis of this magnitude would provide valuable insight into the problem but is beyond the scope of this thesis.

Input Values α, l_i	Signals Generated	Signals Detected	Maximum Likelihood Estimates	Method of Moments Estimates	Least Squares Estimates
$\alpha = 1$					
	25	13	14.45	13.46	15.29
.6/.7/.8	100	70	82.78	79.26	85.37
	1000*	650	769.90	734.99	802.47
	10000	6537	7804.23	7596.42	7971.95
	25	7	9.55	10.67	9.09
.2/.3/.4	100*	40	68.78	64.36	72.73
	1000	378	785.35	771.09	804.26
	10000	3651	7490.23	7330.80	7606.25
	25	14	17.56	19.20	16.67
.2/.5/.8	100	54	66.48	64.57	68.35
	1000	547	750.55	759.06	749.31
	10000	5465	7590.86	7459.31	7697.18
$\alpha = 10$					
	25	24	26.58	29.29	26.67
.6/.7/.8	100	98	101.42	100.26	101.03
	1000	955	999.50	997.43	994.79
	10000*	9522	9951.69	9942.84	9918.75
	25	21	27.76	27.90	27.63
.2/.3/.4	100	62	84.35	81.73	86.11
	1000	601	982.82	1007.44	969.35
	10000	6262	10184.48	10208.45	10100.00
	25*	19	24.94	23.63	26.03
.2/.5/.8	100	86	99.18	98.59	100.00
	1000	864	990.72	974.08	1004.65
	10000	8809	10020.67	10043.02	10010.23
$\alpha = 100$					
	25	24	24.79	24.32	24.74
.6/.7/.8	100	98	100.96	100.97	101.03
	1000	980	1004.61	1009.47	1010.31
	10000	9720	9988.40	9973.44	10020.62
	25*	20	42.46	36.00	51.28
.2/.3/.4	100	61	85.26	90.56	82.43
	1000	651	948.93	950.72	943.48
	10000	6539	9887.67	9830.35	9907.57
	25	25	27.40	28.21	26.88
.2/.5/.8	100	93	100.46	99.64	101.09
	1000*	906	995.26	984.95	1006.67
	10000	9125	9972.98	9965.46	10027.47

Table 1 • Estimates of Signals Generated/3-Receiver System

Input Values α, l_i	Signals Generated	Signals Detected	Maximum Likelihood Estimates	Method of Moments Estimates	Least Squares Estimates
$\alpha = 1$					
	25*	19	19.40	18.74	19.59
.6/.7/.8	100	72	73.95	69.83	76.60
	1000	773	803.01	716.65	831.18
	10000	7965	8248.49	7510.09	8384.21
	25	14	14.98	14.62	15.56
.2/.3/.4	100	49	61.17	58.70	62.03
	1000	566	747.70	698.84	754.67
	10000	5614	7886.43	7512.48	7907.04
	25	17	20.02	21.25	19.54
.2/.5/.8	100	70	75.01	74.83	76.92
	1000	719	796.05	751.39	798.89
	10000*	7344	8104.96	7606.72	8251.68
$\alpha = 10$					
	25	25	25.04	24.87	25.00
.6/.7/.8	100	100	100.23	99.54	100.00
	1000	996	997.79	993.13	996.00
	10000	9969	9989.28	9955.48	9969.00
	25	24	27.90	27.83	28.57
.2/.3/.4	100*	91	104.85	105.35	105.81
	1000	844	977.51	978.60	981.40
	10000	8568	9958.32	9866.30	9962.79
	25	24	24.33	24.44	27.27
.2/.5/.8	100	97	98.40	99.23	98.98
	1000*	990	1006.67	995.82	1010.20
	10000	9811	9956.08	9924.80	9910.10
$\alpha = 100$					
	25	25	25.01	24.85	26.04
.6/.7/.8	100*	99	99.07	97.91	99.00
	1000	998	998.94	997.17	998.00
	10000	9994	10000.83	9992.41	9994.00
	25	22	25.42	23.06	26.51
.2/.3/.4	100	79	86.98	94.81	86.81
	1000	876	985.43	978.01	995.45
	10000*	8777	9807.58	9893.83	9973.86
	25	24	24.12	24.93	24.00
.2/.5/.8	100	9	99.79	100.70	101.02
	1000	999	1006.50	1007.47	999.00
	10000	9926	10002.23	9976.55	9926.00

Table 2 • Estimates of Signals Generated/6-Receiver System (Two receivers share a common intercept probability)

ALFA = 1

RECEIVER PROB VECTOR	MAXIMUM LIKELIHOOD ESTIMATES				METHOD OF MOMENTS ESTIMATES				LEAST SQUARES ESTIMATES			
	MEAN	VAR	MIN	MAX	MEAN	VAR	MIN	MAX	MEAN	VAR	MIN	MAX
NUMBER OF RECEIVERS: 3					NUMBER OF SIGNALS GENERATED: 100							
.10	.1257	.0027			.1404	.0037			.1217	.0036		
.15	.1949	.0048			.2011	.0062			.1769	.0072		
.20	.2536	.0080			.2707	.0099			.2509	.0120		
(MAE)	.1951	.0119	.0260	.5239	.2267	.0205	.0074	.7551	.2161	.0158	.0237	.5572
NUMBER OF RECEIVERS: 3					NUMBER OF SIGNALS GENERATED: 1000							
.30	.3896	.0003			.4004	.0003			.3814	.0004		
.35	.4542	.0005			.4666	.0006			.4403	.0004		
.40	.5197	.0006			.5335	.0005			.5085	.0004		
(MAE)	.3135	.0022	.1557	.4261	.3505	.0021	.2285	.4635	.2801	.0016	.1799	.3989
NUMBER OF RECEIVERS: 6					NUMBER OF SIGNALS GENERATED: 25							
.30	.3866	.0116			.3911	.0181			.2862	.0284		
.30	.3432	.0097			.3895	.0160			.2749	.0272		
.35	.4124	.0136			.4452	.0127			.3384	.0360		
.35	.4255	.0122			.4568	.0175			.3403	.0375		
.40	.4669	.0115			.5399	.0161			.3970	.0461		
.40	.4949	.0126			.5208	.0177			.3895	.0465		
(MAE)	.6333	.0454	.2193	1.265	.8114	.0737	.2569	1.509	.9353	.3078	.1832	1.882
NUMBER OF RECEIVERS: 6					NUMBER OF SIGNALS GENERATED: 10000							
.10	.1123	.0002			.1328	10-5			.1220	10-5		
.10	.1125	.0001			.1333	10-5			.1215	10-5		
.25	.3030	.0001			.3333	10-5			.3052	10-5		
.25	.3016	.0002			.3331	10-5			.3058	10-5		
.40	.5041	10-4			.5333	10-5			.4875	10-5		
.40	.5051	10-4			.5330	10-5			.4882	10-5		
(MAE)	.3853	.0016	.2703	.4593	.4987	.0003	.4604	.5376	.3301	.0003	.2881	.3748

TABLE 3. MEAN AND VARIANCE ON ESTIMATES OF RECEIVER INTERCEPT PROBABILITIES. (MEAN ABSOLUTE ERROR ESTIMATES ARE COMPUTED OVER THE ENTIRE PROBABILITY VECTOR.)

ALFA = 10

RECEIVER PROB VECTOR	MAXIMUM LIKELIHOOD ESTIMATES MEAN	VAR	MIN	MAX	METHOD OF MOMENTS ESTIMATES MEAN	VAR	MIN	MAX	LEAST SQUARES ESTIMATES MEAN	VAR	MIN	MAX
NUMBER OF RECEIVERS: 3			NUMBER OF SIGNALS GENERATED: 25									
.18	.1902	.0065			.1790	.0060			.1625	.0071		
.45	.4529	.0143			.4517	.0121			.4397	.0258		
.73	.7432	.0170			.7484	.0181			.6798	.0516		
(MAE)	.2620	.0157	.0409	.5955	.2563	.0152	.0435	.5636	.3305	.0954	.0441	1.223
NUMBER OF RECEIVERS: 3			NUMBER OF SIGNALS GENERATED: 10000									
.55	.5482	10-5			.5491	10-5			.5494	10-5		
.64	.6405	10-5			.6420	10-5			.6418	10-5		
.73	.7304	10-5			.7331	10-5			.7336	10-5		
(MAE)	.0147	10-4	.0021	.0410	.0181	10-4	.0020	.0428	.0196	10-4	.0094	.0330
NUMBER OF RECEIVERS: 6			NUMBER OF SIGNALS GENERATED: 100									
.18	.1824	.0018			.1823	.0021			.1839	.0016		
.18	.1873	.0017			.1855	.0017			.1833	.0014		
.27	.2762	.0026			.2763	.0027			.2739	.0021		
.27	.2768	.0025			.2719	.0019			.2757	.0025		
.36	.3680	.0029			.3648	.0025			.3662	.0035		
.36	.3635	.0023			.3692	.0030			.3632	.0026		
(MAE)	.2327	.0066	.1195	.4615	.2253	.0081	.0622	.6506	.2483	.0037	.1063	.3577
NUMBER OF RECEIVERS: 6			NUMBER OF SIGNALS GENERATED: 1000									
.18	.1922	.0002			.1859	.0001			.1807	.0001		
.18	.1918	.0002			.1829	.0001			.1819	.0001		
.45	.4600	.0003			.4587	.0002			.4548	.0003		
.45	.4599	.0002			.4592	.0002			.4587	.0002		
.73	.7201	.0003			.7346	.0002			.7295	.0003		
.73	.7218	.0002			.7346	.0002			.7396	.0002		
(MAE)	.0810	.0005	.0394	.1311	.0687	.0003	.0351	.1158	.0693	.0004	.0305	.1394

TABLE 4. MEAN AND VARIANCE ON ESTIMATES OF RECEIVER INTERCEPT PROBABILITIES. (MEAN ABSOLUTE ERROR ESTIMATES ARE COMPUTED OVER THE ENTIRE PROBABILITY VECTOR.)

ALFA = 100

RECEIVER PROB VECTOR	MAXIMUM LIKELIHOOD ESTIMATES MEAN	VAR	MIN	MAX	METHOD OF MOMENTS ESTIMATES MEAN	VAR	MIN	MAX	LEAST SQUARES ESTIMATES MEAN	VAR	MIN	MAX
NUMBER OF RECEIVERS: 3				NUMBER OF SIGNALS GENERATED: 25								
.20	.2063	.0087			.2061	.0102			.1740	.0103		
.30	.3041	.0147			.2942	.0173			.2621	.0188		
.40	.3826	.0168			.3953	.0205			.3541	.0221		
(MAE)	.2800	.0185	.0387	.6619	.2962	.0239	.0587	.7298	.3100	.0403	.0342	.7711
NUMBER OF RECEIVERS: 3				NUMBER OF SIGNALS GENERATED: 1000								
.20	.1980	.0001			.1980	.0001			.1967	.0002		
.50	.4943	.0003			.4953	.0004			.4949	.0003		
.79	.7895	.0002			.7916	.0004			.7917	.0004		
(MAE)	.0350	.0003	.0030	.0828	.0393	.0004	.0040	.1015	.0414	.0003	.0118	.0939
NUMBER OF RECEIVERS: 6				NUMBER OF SIGNALS GENERATED: 100								
.59	.5967	.0028			.5899	.0023			.5876	.0022		
.59	.5940	.0025			.5953	.0025			.5874	.0021		
.69	.6987	.0021			.6902	.0029			.6874	.0022		
.69	.6872	.0024			.6987	.0017			.6856	.0020		
.79	.7877	.0025			.7940	.0016			.7901	.0017		
.79	.7914	.0017			.7929	.0015			.7830	.0014		
(MAE)	.2279	.0070	.1026	.4638	.2239	.0051	.0688	.4079	.2147	.0042	.0532	.3500
NUMBER OF RECEIVERS: 6				NUMBER OF SIGNALS GENERATED: 10000								
.20	.2114	10-4			.1984	10-5			.1983	10-5		
.20	.2101	10-5			.1987	10-5			.1984	10-5		
.30	.3070	10-4			.2976	10-5			.2966	10-5		
.30	.3076	.0001			.2971	10-5			.2972	10-5		
.40	.4012	10-5			.3960	10-5			.3956	10-5		
.40	.4016	10-5			.3959	10-5			.3965	10-5		
(MAE)	.0617	.0003	.0210	.0960	.0231	10-4	.0009	.0474	.0244	10-4	.0093	.0458

TABLE 5. MEAN AND VARIANCE ON ESTIMATES OF RECEIVER INTERCEPT PROBABILITIES. (MEAN ABSOLUTE ERROR ESTIMATES ARE COMPUTED OVER THE ENTIRE PROBABILITY VECTOR.)

ALFA = 1

N NUMBER SIGNALS GENERATED
K NUMBER RECEIVERS
PI RECEIVER PROB VECTOR

Case		Statistic	MAXIMUM LIKELIHOOD ESTIMATES	METHOD OF MOMENTS ESTIMATES	LEAST SQUARES ESTIMATES
		MEAN	88.27	83.73	105.60
		VARIANCE	1252	1658	6273
N = 100	(MAE)				
K = 3		MEAN	29.93	33.46	46.84
PI = .10/.15/.20		VARIANCE	485.74	795.10	4088.0
		MIN	.10	.80	10-5
		MAX	145.13	257.00	300.00
		MEAN	771.5	753.4	790.0
		VARIANCE	392.2	481.9	563.3
N = 1000	(MAE)				
K = 3		MEAN	228.45	246.65	210.01
PI = .30/.35/.40		VARIANCE	392.19	481.86	563.27
		MIN	165.18	192.03	156.25
		MAX	286.65	289.75	276.19
		MEAN	21.17	19.16	61.59
		VARIANCE	4.54	4.10	5352
N = 25	(MAE)				
K = 6		MEAN	3.86	5.84	42.55
PI = .30/.30/.35/.35/		VARIANCE	4.07	4.10	4875.2
.40/.40		MIN	.04	.81	.53
		MAX	10.78	10.35	195.00
		MEAN	8021	7498	8184
		VARIANCE	3710	3184	4474
N = 10000	(MAE)				
K = 6		MEAN	1979.3	2502.0	1816.2
PI = .10/.10/.25/.25/		VARIANCE	3709.7	3184.2	4477.1
.40/.40		MIN	1824.0	2387.3	1665.9
		MAX	2134.4	2673.2	1966.3

TABLE 6. MEAN AND VARIANCE ON ESTIMATES OF TOTAL SIGNAL POPULATION SIZE. (MEAN ABSOLUTE ERROR ESTIMATES ARE COMPUTED BY COMPARING THE ESTIMATED SIGNAL POPULATION TO THE NUMBER OF SIGNALS GENERATED.)

ALFA = 10

N NUMBER SIGNALS GENERATED
K NUMBER RECEIVERS
PI RECEIVER PROB VECTOR

			MAXIMUM LIKELIHOOD ESTIMATES	METHOD OF MOMENTS ESTIMATES	LEAST SQUARES ESTIMATES
N = 25, K = 3, PI = .18/.45/.73		MEAN	24.69	24.90	43.42
		VARIANCE	9.31	12.50	3156
	(MAE)	MEAN	2.40	2.81	20.28
		VARIANCE	3.58	4.53	3083.2
		MIN	.06	0.0	10-5
		MAX	10.37	10.00	205.00
N = 10000, K = 3, PI = .55/.64/.73		MEAN	9945	9915	9911
		VARIANCE	632.5	1154	1482
	(MAE)	MEAN	55.19	84.75	91.61
		VARIANCE	571.95	1154.0	994.46
		MIN	5.77	6.91	5.26
		MAX	126.88	164.01	153.13
N = 100, K = 6, PI = .18/.18/.27/.27/.36/.36		MEAN	99.54	100.70	99.77
		VARIANCE	31.00	45.14	28.61
	(MAE)	MEAN	4.46	5.50	4.04
		VARIANCE	11.14	15.09	12.16
		MIN	.14	0.0	10-5
		MAX	15.08	15.61	15.91
N = 1000, K = 6, PI = .18/.18/.45/.45/.73/.73		MEAN	997.3	991.5	996.8
		VARIANCE	23.07	59.55	42.66
	(MAE)	MEAN	4.29	9.41	5.91
		VARIANCE	12.04	43.61	17.85
		MIN	.05	10-3	10-4
		MAX	15.20	30.73	16.16

TABLE 7. MEAN AND VARIANCE ON ESTIMATES OF TOTAL SIGNAL POPULATION SIZE. (MEAN ABSOLUTE ERROR ESTIMATES ARE COMPUTED BY COMPARING THE ESTIMATED SIGNAL POPULATION TC THE NUMBER OF SIGNALS GENERATED.)

ALFA = 100

N NUMBER SIGNALS GENERATED
K NUMBER RECEIVERS
PI RECEIVER PROB VECTOR

			MAXIMUM LIKELIHOOD ESTIMATES	METHOD OF MOMENTS ESTIMATES	LEAST SQUARES ESTIMATES
N = 25		MEAN	27.43	27.61	42.32
K = 3		VARIANCE	116.2	112.2	1833
PI = .20/.30/.40	(MAE)	MEAN	6.69	6.80	20.47
		VARIANCE	76.98	72.33	1712.8
		MIN	.04	0.0	10-5
		MAX	46.86	65.00	175.00
N = 1000		MEAN	1001	1001	1002
K = 3		VARIANCE	139.0	270.1	235.3
PI = .20/.50/.79	(MAE)	MEAN	9.23	12.74	12.59
		VARIANCE	54.20	107.91	78.35
		MIN	.15	.09	10-4
		MAX	32.66	51.92	42.22
N = 100		MEAN	100.00	100.00	100.6
K = 6		VARIANCE	.03	1.27	.72
PI = .59/.59/.69/.69/.79/.79	(MAE)	MEAN	.10	.86	.60
		VARIANCE	.02	.52	.70
		MIN	.01	0.0	10-5
		MAX	.98	3.34	3.09
N = 10000		MEAN	9901	10000	10000
K = 6		VARIANCE	2834	3620	4811
PI = .20/.20/.30/.30/.40/.40	(MAE)	MEAN	101.83	48.41	60.45
		VARIANCE	2330.3	1254.2	1123.2
		MIN	1.57	1.25	1.13
		MAX	202.83	142.37	128.09

TABLE 8. MEAN AND VARIANCE ON ESTIMATES OF TOTAL SIGNAL POPULATION SIZE. (MEAN ABSOLUTE ERROR ESTIMATES ARE COMPUTED BY COMPARING THE ESTIMATED SIGNAL POPULATION TO THE NUMBER OF SIGNALS GENERATED.)

MEAN ABSOLUTE ERROR SCORES
ON ESTIMATES OF RADIO INTERCEPT PROBABILITIES

		Maximum Likelihood Estimates	Method of Moments Estimates	Least Squares Estimates
Alpha = 1	ΣE(MAE)	1.5272	1.8873	1.7616
	ΣVar(MAE)	.0611	.0993	.3255
Alpha = 10	ΣE(MAE)	.5904	.5684	.6677
	ΣVar(MAE)	.0229	.0237	.0996
Alpha = 100	ΣE(MAE)	.6046	.5822	.5905
	ΣVar(MAE)	.0261	.0295	.0449

MEAN ABSOLUTE ERROR SCORES
ON ESTIMATES OF TOTAL SIGNAL POPULATION SIZE

		Maximum Likelihood Estimates	Method of Moments Estimates	Least Squares Estimates
Alpha = 1	ΣE(MAE)	2241.54	2787.95	2115.60
	ΣVar(MAE)	4591.70	4465.26	14003.57
Alpha = 10	ΣE(MAE)	66.34	102.47	121.84
	ΣVar(MAE)	598.71	1217.23	4107.67
Alpha = 100	ΣE(MAE)	117.85	68.81	94.11
	ΣVar(MAE)	2461.50	1437.97	2915.05

Table 9. Mean Absolute Error Scores

VI. SUMMARY AND RECOMMENDATIONS

This thesis has presented a development of various estimators which can be used to estimate the total number of signals passing through an acquisition system given only a portion were detected. Likewise, estimators were found for estimating the intercept probabilities for each receiver in the system. These estimators were used in the context of a model developed to represent the signal detection process. The model suggested is the foundation for a simulation conducted over a variety of conditions including receiver and signal characteristics as well as transmitter activity. The simulation results provided some general insight into the quality of the estimators developed and their behavior over varying conditions. The question of dependence between receivers was briefly addressed and it was shown that as dependency increased, the quality of the estimates decreased.

The work presented here is not viewed as a final solution to the problem. The model used was designed for flexibility and it would be a simple matter to refine and expand the detection process used within this report. The simulation conducted was sufficient for validating the model and obtaining some initial insight into the quality of the estimators but was not adequate for making conclusive claims

about them. It is hoped that this thesis may stimulate others into continuing work on this subject. In particular, it is suggested that a complete experiment be designed and a formal data analysis of the simulation results be conducted to determine which method of estimation is optimal. It would equally be interesting if someone with an electronics background could incorporate into the model additional parameters or functional relationships bringing in some hardware considerations or radiowave characteristics.

APPENDIX A

SIMULATION PACKAGE CHARACTERISTICS AND REQUIREMENTS

The simulation discussed in section IV is designed to minimize user interaction. The user must decide what is desired to be accomplished by the simulation and modify the main routine and output routine accordingly. The material discussed in this appendix suggests simple instructions on how to use the FORTRAN simulation package provided with this study. A complete source listing of the FORTRAN code is located at the end of this appendix.

A. CHARACTERISTICS

As currently designed, the simulation is dimensioned to handle an acquisition system of up to ten receivers. The total number of signals which can be generated in any one replication is 99,999 , limited only by the input format statement allowing five digits. Similarly, the random number generator seed is formatted for up to five digits. To modify the simulation to handle a larger acquisition system or increase the number of signals desired to be generated is a simple matter of increasing the dimensionality of the arrays and enlarging input formats.

B. METHOD SELECTION

The user must determine which one or more methods of estimation to be performed. Should only one method of estimation be desired, that method is requested through a coded input parameter in the data base as follows:

Maximum Likelihood....METHOD = 1

Method of Moments.....METHOD = 2

Least Squares.........METHOD = 3.

If more than one method is desired, the user should provide the lowest coded input parameter in the data base and establish the appropriate looping in the main routine. The loop established in the source listing at the end of this appendix provides for all three methods to be applied to a common signal data set created by subroutine GNRATE.

C. REQUIREMENTS WITH THE USE OF GRG

If the user specifies the maximum likelihood method of estimation to be used, the user supplied subroutine GCOMP must be included in the source listing. This subroutine contains the objective function to which the generalized reduced gradient (GRG) routine attempts to maximize. (In actuality, the GRG routine minimizes an objective function, thus, to maximize the log-likelihood function it is necessary to minimize the negative of the objective function.) Subroutine GCOMP contained in the source listing provided is general in nature and accurately represents the log-

likelihood function discussed in section III for any k-receiver system. No user modifications to this routine are necessary for systems of ten receivers or less.

Each time the maximum likelihood method is used during the course of a simulation, the GRG routine is called. With each call to subroutine GRG, it is necessary to provide a special input data base to support the optimization. This data base is read by library routine DATAIN, and is supplementary to the initial data base established for the simulation.

D. DATA INPUT

Six data cards initialize the simulation. An additional set of data cards are necessary for each call to subroutine GRG. The size of the data set required for GRG is related to the number of receivers. Shown at Figure 5 is a sample data set for a three receiver system. The data is preceeded by a data section number and followed by the input format. Below, is a description of each data entry by data section number.

Data Sctn Number	Data Elements and Description
1	Random number generator seed (read once).
2	Number of signals to be generated.
3	Number of receivers in the system.
4	Input receiver intercept probability vector.

5	Value of alpha used in generating $B(\alpha, 1)$ variates.
6	Method of analysis to be used.
7	Problem name. Any combination of 80 alphanumeric characters.
8	Number of receivers (values of π_i to be estimated) followed by a '1' and '0', the number of equality and inequality constraints in the maximization.
9	Number of variables with finite lower bounds (equal to the number of receivers).
10	There should be k cards each containing a pair of numbers. The first number is the index of the variable in 9 above followed by its lower bound. In this simulation the lower bound is set at .0005 .
11	Number of variables with finite upper bounds (again, equal to the number of receivers).
12	There should be k cards each containing a pair of numbers. The first number is the index of the variable in 11 above followed by its upper bound. In this simulation the upper bound is set at .9999.
13	Number of inequality constraints having finite upper bounds. Enter 0.
14	Initial values for π_i. Enter .50 for each.
15	Tolerance value when certain iterative processes should stop. Default value is zero.
16	Limits on the upper bound of allowed iterations. Default value is zero.
17	Specification on the desired amount of output. Zero is the default value and

a value of one allows the user to increase or decrease output. A value of one is used to suppress output in this simulation.

18 A pair of zero values effects the suppression of output.

19 Anything but 'QUAD' will assure the use of tangent vector extrapolation for estimating initial values of basic variables.

20 If the user does not wish to partially or completely specify an initial basis, enter a zero on this card.

For a more detailed discussion on GRG refer to a GRG user's guide (i.e. see Lasdon, 1975).

E. OUTPUT SELECTION

Two forms of subroutine OUTPUT are provided with the source listing. The first form is used when one or more environments are specified and replicated once each. This form would typically be used with actual field data. The second form of subroutine OUTPUT is used when it is desired to replicate one environment a number of times to allow for a statistical analysis of the estimators $\hat{\pi}_i$ and $\hat{s}$. The user need only to decide which form of the output is applicable to the simulation being conducted and insert the appropriate listing.

Data Section Number	Sample Data	Format
	Card Column 123456789111111111122222222223333333333 0123456789012345678901234567890	
1	21215	I5
2	10000	I5
3	3	I5
4	.60 .70 .80	10F5.2
5	100.	F5.1
6	1	I5
7	Problem name. Any 80 Alphanumerics.	20A4
8	3 1 0	3I5
9	3	I5
10	1 .0005	I5,5X,E10.4
	2 .0005	
	3 .0005	
11	3	I5
12	1 .9999	I5,5X,E10.4
	2 .9999	
	3 .9999	
13	0	I5
14	.50 .50 .50	8E10.4
15	0	I5
16	0	I5
17	1	I5
18	0 0	2I5
19	0	A4
20	0	I5

Figure 5 • Sample Data Set for a 3-Receiver System

```
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ARRAY, B, C, START, STOP
      DIMENSION ARRAY(10,1024), C(1024), JSUM(1024), G(2), X(10),
     1 PSUB(11), PHAT(11), PTWT(11), Q(11), COEF(11)
      CCMMON /GRP1/ N, K, KCELL, METHOD
      CCMMON /GRP2/ TAU(10), ALFA
      COMMON /GRP3/ SUM, XKPLS1,SMXIXJ, A(10), D(10), R(10)
      CCMMON /GRP4/ SYSTMP, TAUHAT(10), CAPN
C
      ITRIAL = 1
      READ (5,100) ISEED
10    READ (5,100) N
      READ (5,100) K
      READ (5,110) (TAU(I), I=1,K)
      READ (5,120) ALFA
      READ (5,130) METHOD
C
      CALL GNRATE (C,ISEED)
      CALL COUNT (C)
90    CALL SOLVE
      CALL OUTPUT (C)
C
      METHOD = METHOD + 1
      IF (METHOD.GT.3) GO TO 95
      GO TO 90
95    ITRIAL = ITRIAL + 1
      IF (ITRIAL.EQ.73) GO TO 140
      GC TO 10
C
100   FORMAT (I5)
110   FORMAT (10F5.2)
120   FCRMAT (F5.1)
130   FORMAT (I5)
C
140   STOP
      END


      SUBROUTINE GNRATE (C,ISEED)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL U, W
      INTEGER B,C
      DIMENSION C(1024)
      CCMMON /GRP1/ N, K, KCELL, METHOD
      CCMMON /GRP2/ TAU(10), ALFA
C
      KCELL = 2**K
```

```
C
      DC 10 J = 1,KCELL
         C(J) = 0
10    CONTINUE
      DO 30 L = 1,N
         B = 1
         CALL LRND (ISEED,W,1,M1,0)
         V = W**(1./ALFA)
         DO 20 I = 1,K
               CALL LRND (ISEED, U, 1, M1, 0)
               FTAU = TAU(I) * V
               IF ( U.LE.FTAU ) B = B + 2**(I-1)
20       CONTINUE
         C(B) = C(B) + 1
30    CONTINUE
C
      RETURN
      END


      SUBROUTINE COUNT (C)
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ARRAY, C, START, STOP
      DIMENSION ARRAY(10,1024), C(1024), JSUM(1024),MULT(1024)
      CCMMON /GRP1/ N, K, KCELL, METHOD
      CCMMON /GRP3/ SUM, XKPLS1, SMXIXJ, A(10), D(10), R(10)
C
      SUM = 0.0
      DC 20 I = 1,K
         DO 10 J = 1,KCELL
               ARRAY(I,J) = 0
               IF (I.EQ.1.AND.J.GE.2) SUM = SUM + C(J)
10       CONTINUE
20    CONTINUE
C
      M = K
      DO 50 I = 1,K
         A(I) = 0.0
         INDEX = 2**(I-1)
         START = INDEX + 1
         STOP = 2*INDEX
30       DO 40 J = START,STOP
               ARRAY(M,J) = 1
               A(I) = A(I) + C(J)
40       CONTINUE
         D(I) = SUM - A(I)
         START = STOP + INDEX + 1
```

```
         STOP = START + INDEX - 1
         IF (START.GT.KCELL) GO TO 45
         GO TO 30
45       M = M - 1
50    CONTINUE
C
      XKPLS1 = 0.0
      JSUM(1) = 0
      MULT(1) = 0
      DO 80 J = 2,KCELL
         JSUM(J) = 0
         MULT(J) = 0
         DO 60 I = 1,K
            JSUM(J) = JSUM(J) + ARRAY(I,J)
60       CONTINUE
         MM = JSUM(J) - 1
         IF (MM.EQ.0) GO TO 70
         DO 70 M = 1,MM
            MULT(J) = MULT(J) + M
70       CONTINUE
         XKPLS1 = XKPLS1 + MULT(J)*C(J)
80    CONTINUE
C
      STOP = K-1
      SMXIXJ = 0.0
      DO 100 I = 1,STOP
         M = I+1
         DO 90 J = M,K
            SMXIXJ = SMXIXJ + A(I)*A(J)
90       CONTINUE
100   CONTINUE
C
      DO 120 I=1,K
         R(I) = 0.0
         DO 110 J = 1,KCELL
            IF (JSUM(J).EQ.I) R(I) = R(I) + C(J)
110      CONTINUE
120   CONTINUE
C
C
      RETURN
      END


      SUBROUTINE SOLVE
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER STOP
```

```
      DIMENSION PSUB(11),PHAT(11),PTWT(11),Q(11),COEF(11)
      CCMMON /GRP1/ N,K,KCELL,METHOD
      CCMMON /GRP2/ TAU(10), ALFA
      COMMON /GRP3/ SUM, XKPLS1, SMXIXJ, A(10), D(10), R(10)
      CCMMON /GRP4/ SYSTMP, TAUHAT(10), CAPN
C
      IF (METHOD.EQ.1) GO TO 10
      IF (METHOD.EQ.2) GO TO 30
      IF (METHOD.EQ.3) GO TO 50
C
C     **************************************************************
C     ***                                                        ***
C     ***      METHOD 1:  MAXIMUN LIKELIHOOD/                    ***
C     ***                 CONDITIONAL MAXIMUM LIKELIHOOD         ***
C     ***                                                        ***
C     **************************************************************
C
10    CALL DATAIN
      CALL GRG
      CALL OUTRES
C
C
      GBG = 1.0
      DO 20 I = 1,K
         DIFF = 1. - TAUHAT(I)
         GBG = GBG*DIFF
20    CONTINUE
      CAPN = SUM/(1. - GBG)
      GO TO 150
C
C     **************************************************************
C     ***                                                        ***
C     ***      METHOD 2:  METHOD OF MOMENTS                      ***
C     ***                                                        ***
C     **************************************************************
C
30    CAPN = SMXIXJ/XKPLS1
      DC 40 I = 1,K
         TAUHAT(I) = A(I)*XKPLS1/SMXIXJ
40    CONTINUE
      GC TO 150
C
C     **************************************************************
C     ***                                                        ***
C     ***      METHOD 3:  LEAST SQUARES                          ***
C     ***                                                        ***
C     **************************************************************
C
50    P = .20
```

```
      ICOUNT = 1
      TEMP = 1.0
51    DO 55 I=1,K
         PSUB(I) = (A(I)/SUM) * P
         Q(I) = PSUB(I)/(1.-PSUB(I))
55    CONTINUE
C
      COEF(1) = 1.0
      DO 70 I=1,K
         STOP = I+1
         L = STOP
         COEF(L) = 0.0
         DO 60 J=2,STOP
               COEF(L) = COEF(L) + COEF(L-1) * Q(I)
               L = L-1
60       CONTINUE
70    CONTINUE
C
      STOP = K+1
      S = 0.0
      DO 80 I=1,STOP
         S = S + COEF(I)
80    CONTINUE
      DC 90 I=1,STOP
         PTWT(I) = COEF(I)/S
90    CONTINUE
C
      PHAT(1) = 1. - P
      DC 100 I = 1,K
         PHAT(I+1) = (R(I)/SUM) * P
100   CONTINUE
C
      EP = 0.0
      DO 110 I=1,STOP
         EP = EP + (PHAT(I)-PTWT(I))**2
110   CONTINUE
      IF (EP.LT.TEMP) TEMP = EP
      IF (TEMP.EQ.EP) SYSTMP = P
      IF (ICOUNT.GE.81) GO TO 120
      ICOUNT = ICOUNT + 1
      P = P + .01
      GO TO 51
C
120   DO 130 I=1,K
         TAUHAT(I) = (A(I)/SUM) * SYSTMP
130   CONTINUE
C
      CAPN = SUM/SYSTMP
```

```
C
150    RETURN
       END


       SUBROUTINE GCOMP (G,X)
       IMPLICIT REAL*8 (A-H,O-Z)
       DIMENSION G(2), X(10)
       COMMON /GRP1/ N, K, KCELL, METHOD
       COMMON /GRP3/ SUM, XKPLS1, SMXIXJ, A(10), D(10), R(10)
       CCMMON /GRP4/ SYSTMP, TAUHAT(10), CAPN
C
       G(1) = 1.0
       TEMP1 = 0.0
       TEMP2 = 1.0
       DO 10 I = 1,K
          TEMP1 = TEMP1 + A(I)*DLOG10(X(I)) + D(I)*DLOG10(1.-X(I))
          TEMP2 = TEMP2*(1.-X(I))
          TAUHAT(I) = X(I)
10     CONTINUE
       G(2) = -(TEMP1 - SUM*DLOG10(1.-TEMP2))
C
       RETURN
       END

(SUBROUTINE OUTPUT FORM 1)

       SUBROUTINE OUTPUT (C)
       IMPLICIT REAL*8 (A-H,O-Z)
       INTEGER C
       DIMENSION C(1024)
       CCMMON /GRP1/ N, K, KCELL, METHOD
       CCMMON /GRP2/ TAU(10), ALFA
       CCMMON /GRP3/ SUM, XKPLS1, SMXIXJ, A(10), D(10), R(10)
       CCMMON /GRP4/ SYSTMP, TAUHAT(10), CAPN
C
       IF (METHOD.NE.1) GO TO 20
       WRITE (6,100)
       WRITE (6,101) N
       WRITE (6,102) K
       WRITE (6,103) ALFA
       WRITE (6,104) (TAU(I), I=1,K)
       WRITE (6,110)
       IF (K.GE.6) GO TO 10
       DO 10 J = 2,KCELL
```

```
            INDEX = J-1
            WRITE (6,111) INDEX, C(J)
10      CONTINUE
        WRITE (6,115)
        WRITE (6,116) (A(I), I=1,K)
        WRITE (6,117) (D(I), I=1,K)
        WRITE (6,118) (R(I), I=1,K)
        WRITE (6,119) SUM
20      CONTINUE
        IF (METHOD.EQ.1) WRITE (6,120)
        IF (METHOD.EQ.2) WRITE (6,121)
        IF (METHOD.EQ.3) WRITE (6,122)
        IF (METHOD.EQ.3) WRITE (6,123) SYSTMP
        WRITE (6,126) (TAUHAT(I), I=1,K)
        WRITE (6,127) CAPN
        IF (METHOD.EQ.3) WRITE (6,128)
C
100     FORMAT ('1','INPUT PARAMETERS TO SIGNAL GENERATOR')
101     FORMAT ('0', 10X,'NUMBER OF SIGNALS GENERATED:', I8)
102     FORMAT (' ', 10X,'NUMBER OF RECEIVERS:',14X,I2)
103     FORMAT (' ',10X,'ALFA PARAMETER:',18X,F5.1)
104     FORMAT (' ',10X,'RECEIVER PROBABILITY VECTOR:',5X,10F5.2)
110     FORMAT ('0',//,' CELL COUNTS',/)
111     FORMAT (' ',12X,'N(',I3,') = ',I4)
115     FORMAT ('0',//,' OPTIMIZATION COEFFICIENTS')
116     FORMAT ('0',5X,'OBSERVED NUMBER OF DETECTIONS BY RECIEVER I:',//,
       1 10X, 10F8.1)
117     FORMAT ('0',5X,'OBSERVED NUMBER OF NO-DETECTS BY RECIEVER I:',//,
       1 10X, 10F8.1)
118     FORMAT ('0',5X,'OBSERVED NUMBER OF TIMES J RECIEVERS DETECTED A CO
       1MMON SIGNAL:',//, 10X, 10F8.1)
119     FORMAT ('0',5X,'NUMBER OF SIGNALS INTERCEPTED:',F10.1)
120     FORMAT('0',///,' OPTIMIZATION METHOD:  MAXIMUM LIKELIHOOD/CONDITIO
       1NAL MAXIMUM LIKELIHOOD')
121     FORMAT ('0',///,' OPTIMIZATION METHOD:  METHOD OF MOMENTS')
122     FORMAT ('0',///,' OPTIMIZATION METHOD:  LEAST SQUARES')
123     FORMAT ('0',/,5X, 'SYSTEM INTERCEPT PROBABILITY:',5X, F5.2)
126     FORMAT ('0',/,5X, 'ESTIMATES OF INTERCEPT PROBABILITIES:',//,
       1 10X,10F5.2)
127     FORMAT ('0',/,5X,'ESTIMATE OF THE TOTAL NUMBER OF TRANSMISSIONS:',
       1 F10.2)
128     FORMAT ('1')
C
        RETURN
        END
```

(SUBROUTINE OUTPUT FORM 2)

```
      SUBROUTINE OUTPUT
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL MAEN, MAEP
      DIMENSION TEMPN(100), TEMPP(10,100),SUMP(10), EVALP(10),SSQP(10),
     1 VARP(10)
      CCMMON /GRP1/ N,K,KCELL,METHOD
      CCMMON /GRP2/ TAU(10), ALFA
      CCMMON /GRP4/ SYSTMP, TAUHAT(10), CAPN
      COMMON /GRP5/ MAEN(100), MAEP(100)
      COMMON /GRP6/ M
C
      TEMPN(M) = CAPN
      DO 10 I =1,K
         TEMPP(I,M) = TAUHAT(I)
10    CONTINUE
C
      IF (M.NE.1) GO TO 30
      SUMN = 0.0
      DC 20 I =1,K
         SUMP(I) = 0.0
20    CONTINUE
C
30    MAEP(M) = 0.0
      DC 40 I = 1,K
         MAEP(M) = MAEP(M) + DABS(TAUHAT(I)-TAU(I))
40    CONTINUE
      MAEN(M) = DABS(CAPN - N)
C
      SUMN = SUMN + CAPN
      DO 50 I =1,K
         SUMP(I) = SUMP(I) + TAUHAT(I)
50    CONTINUE
C
      IF (M.LT.100) GO TO 250
C
      EVALN = SUMN/FLOAT(M)
      DO 60 I = 1,K
         EVALP(I) = SUMP(I)/FLOAT(M)
60    CONTINUE
C
      SSQN = 0.0
      DO 70 J = 1,M
         SSQN = SSQN + (TEMPN(J) - EVALN)**2
70    CONTINUE
      VARN = SSQN/FLOAT(M-1)
```

```
C
      DC 90 I = 1,K
         SSQP(I) = 0.0
         DO 80 J = 1,M
            SSQP(I) = SSQP(I) + (TEMPP(I,J) - EVALP(I))**2
80       CONTINUE
90    CONTINUE
      DC 100 I = 1,K
         VARP(I) = SSQP(I)/FLOAT(M-1)
100   CONTINUE
C
      CALL SECTN(MAEP,M,10)
      WRITE (6,208)
      WRITE (6,200)
      WRITE (6,201) N
      WRITE (6,202) K
      WRITE (6,203) ALFA
      WRITE (6,204) (TAU(I), I=1,K)
      IF (METHOD.EQ.1) WRITE (6,205)
      IF (METHOD.EQ.2) WRITE (6,206)
      IF (METHOD.EQ.3) WRITE (6,207)
      WRITE (6,212)
      WRITE (6,214) (EVALP(I), I=1,K)
      WRITE (6,213)
      WRITE (6,214) (VARP(I), I=1,K)
      CALL SECTN(MAEN,M,10)
      WRITE (6,209)
      WRITE (6,200)
      WRITE (6,201) N
      WRITE (6,202) K
      WRITE (6,203) ALFA
      WRITE (6,204) (TAU(I), I=1,K)
      IF (METHOD.EQ.1) WRITE (6,205)
      IF (METHOD.EQ.2) WRITE (6,206)
      IF (METHOD.EQ.3) WRITE (6,207)
      WRITE (6,210) EVALN
      WRITE (6,211) VARN
200   FORMAT ('0','INPUT PARAMETERS TO SIGNAL GENERATOR')
201   FORMAT ('0', 10X,'NUMBER OF SIGNALS GENERATED:', I8)
202   FORMAT (' ', 10X,'NUMBER OF RECEIVERS:',14X,I2)
203   FCRMAT (' ',10X,'ALFA PARAMETER:',18X,F5.1)
204   FORMAT (' ',10X,'RECEIVER PROBABILITY VECTOR:',5X,10F5.2)
205   FORMAT('0',/,' OPTIMIZATION METHOD:  MAXIMUM LIKELIHOOD/CONDITIONA
     1L MAXIMUM LIKELIHOOD')
206   FORMAT ('0',/,' OPTIMIZATION METHOD:  METHOD OF MOMENTS')
207   FORMAT ('0',/,' OPTIMIZATION METHOD:  LEAST SQUARES')
208   FORMAT (' ',30X,'STATISTICS FOR MEAN ABSOLUTE ERROR ON RECEIVER PR
     1OBABILITY VECTOR')
```

```
209   FORMAT (' ',30X,'STATISTICS FOR MEAN ABSOLUTE ERROR ON ESTIMATES O
     1F SIGNALS GENERATED')
212   FORMAT ('0',10X,'AVERAGE VALUE OF RECEIVER PROBAILITIES:')
213   FORMAT ('0',10X,'VARIANCE:')
210   FORMAT ('0',10X,'AVERAGE NUMBER OF SIGNALS ESTIMATED:',5X,D10.4)
211   FORMAT ('0',37X,'VARIANCE:',5X,D10.4)
214   FORMAT ('0',20X,10(3X,D10.4))
250   RETURN
      END
```

APPENDIX B

LIST OF SYMBOLS
(alphabetically)

A: the event $(Z \neq 0)$

B: an index of the elements in the vector N

I^j: number of data vectors z for which $z_. = j$

$K = t!/(n_1! \ldots n_{2^k-1})$

k: number of receivers

L(s,p): likelihood of N at n

l_i: proportion of signals of strength V that receiver i would detect

M_i: number of signals detected by receiver i (outcome is m_i)

m^*: $\sum_{i<j}$ (number of signals detected simultaneously by receivers i and j)

N: random vector with outcome n; $N \sim M_{2^k-1}(s,p)$

$n = (n_o, n_1, \ldots, n_{2^k-1})$: vector of n_z's in some specific order

n_z: number of times z S occurs

$p = (p_o, p_1, \ldots, p_{2^k-1})$: vector of p_z's in same order as for n

$p_{z_.}(j)$: mass function of $z_.$

$q_i = p_i/(1 - p_o)$

$r = 2^k - 1$

S: sample space of Z

s: number of signals

$\hat{s}, \hat{\pi}_i$: estimates of s and π_i, respectively

$T_i(v_j) = \pi_{ij} = l_i \cdot v_j$

t: number of signals detected

U: uniform(0,1) random number

V_j: strength of signal j (outcome is v_j)

$Z = (Z_1, \ldots, Z_k)$: vector of Bernoulli detection or non-detection indicators, for a given signal, for all receivers

$z_{\cdot} = \sum_{i=1}^{k} z_i$: number of receivers which detect a given signal

π_{ij}: probability receiver i detects signal j

$\pi_i = E(T_i(V)) = l_i \cdot E(V)$

α: parameter of V_j where $V_j \sim B(\alpha, 1)$

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